Phys. Med. Biol. 58 (2013) 2823-2840

Resolution recovery reconstruction for a Compton camera

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Received 22 August 2012, in final form 26 February 2013 Published 8 April 2013 Online at stacks.iop.org/PMB/58/2823

Abstract

The spatial resolution from Compton cameras suffers from measurement uncertainties in interaction positions and energies. The degree of degradation in spatial resolution is shift-variant (SV) over the field-of-view (FOV) because the imaging principle is based on the conical surface integration. In our study, the shift-variant point spread function (SV-PSF) is derived from point source measurements at various positions in the FOV and is incorporated into the system matrix of a fully three-dimensional, accelerated reconstruction, i.e. the listmode ordered subset expectation maximization (LMOSEM) algorithm, for resolution recovery. Simulation data from point sources were used to estimate SV and asymmetric parameters for Gaussian, Cauchy, and general parametric PSFs. Although little difference in the fitness accuracy between Gaussian and general parametric PSFs was observed, the general parametric model showed greater flexibility over the FOV in shaping the curve between that for Gaussian and Cauchy functions. The estimated asymmetric SV-PSFs were incorporated into the LMOSEM for resolution recovery. For simulation data from a single point source at the origin, all LMOSEM-SV-PSFs improved the spatial resolution by 2.6 times over the standard LMOSEM. For two pointsource simulations, reconstructions also gave a two-fold improvement in spatial resolution and resulted in a greater recovered activity ratio at different positions in the FOV.

(Some figures may appear in colour only in the online journal)

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0031-9155/13/092823+18\$33.00 © 2013 Institute of Physics and Engineering in Medicine Printed in the UK & the USA 2823

Introduction

A Compton camera is a gamma-ray imaging system that adopts electric collimation based on conical surface integration (Singh 1983). Possible applications of these cameras include nuclear medicine imaging of patients, industrial nondestructive tests, space radiation measurements in astronomy, and detecting and locating special nuclear materials in defense and homeland security (Todd *et al* 1974, Al-Ghamdi and Xu 2003, Schönfelder *et al* 1973, Vetter *et al* 2007, Wahl and Zhong 2011). There has always been high interest in developing better Compton cameras not only for enhancing gamma-ray imaging (Singh and Brechner 1990, LeBlanc *et al* 1999, Lee *et al* 2005, Seo *et al* 2008), but also in close-up imaging, radiation therapy monitoring, and multi-tracer imaging (Zhang *et al* 2004, Llosa *et al* 2004, Peterson *et al* 2010, Motomura *et al* 2008, Seo *et al* 2010).

A Compton camera usually consists of two detectors as illustrated in figure 1. Valid events are recorded when an incident gamma-ray photon undergoes Compton scattering in the first detector (scatterer), and loses its remaining energy in the second detector (absorber). Using the measured energies and positions from the scatterer and absorber, a cone is defined with the cone-axis connecting a pair of interaction positions and scattering angle that is determined by the Compton equation (Singh 1983). By back-projecting all valid events into an image space through cone surfaces, we can estimate the distribution of gamma-ray sources.

One of the most challenging technical problems inhibiting the wide use of Compton cameras is their limited spatial resolution; the spatial resolution of Compton camera suffers from measurement uncertainties in the interaction positions and the absorbed energies in scatterer and absorber, which results in an imprecise delineation of the cone surface. Moreover, the resolution degradation is seriously shift-variant (SV) over the field-of-view (FOV) mainly due to the underlying image formation principles based on the conical surface integration. Figure 2 illustrates such a low spatial resolution of Compton camera images reconstructed using conventional simple back-projection (SBP) and listmode ordered subset expectation maximization (LMOSEM) algorithms. Although LMOSEM yields better spatial resolution than SBP, the resolution is poor especially for medical applications.

To overcome the limited spatial resolution of Compton cameras, we have investigated a method to incorporate point spread functions (PSFs) into the accelerated statistical image reconstruction from Compton camera data, yielding better spatial resolution by resolution recovery performed iteratively. We considered three different PSF models to analyze Compton camera data that includes three pairs of scatterer and absorber measurements. A strategy for the Monte Carlo (MC) simulation to estimate parameters for each PSF-based model using point sources placed at various positions over the FOV is also proposed. The relative improvement in the spatial resolution by applying the SV asymmetric PSF model is demonstrated.

Materials and methods

The degradation in the spatial resolution of a Compton camera is directly related to the measurement uncertainties of the interaction energies and positions at the scattering and absorbing detectors. The angular and positional uncertainties affect the construction of the cones from the measurements, as illustrated in figure 3. The error in determining the scattering angles (ω) arises from the limited energy resolution of the two detectors and from Doppler broadening; the angular uncertainty (Δr) produces a thickened conical surface, as shown in figure 3(A). In contrast, the positional uncertainty (Δd) arose from the segmentation of detected positions and it tilts the cone axis as depicted in figure 3(B) (Ordonez *et al* 1999). The thickened and tilted cones resulting from Δr and Δd cause mis-positioning of the measured



Figure 1. Schematic showing a conventional Compton camera setup and cones defined from the measurements. The camera consists of one DSSD scattering detector and four CZT absorbing detectors



Figure 2. Reconstructed images and their spatial resolutions using (A) SBP (FWHM = 32.6 mm) and (B) ordered-subset expectation maximization (OSEM, FWHM = 12.7 mm) methods.

event in the image space, and the degree of degradation of the spatial resolution should be SV over the scanned FOV. In a previous study, we measured the spatial resolutions of seven point sources at different distances from a Compton camera along the normal (x-axis) to the detector planes, as sketched in figure 4(A). Figure 4(B) shows the distance-dependent of the spatial-resolution degradation (Rohe et al 1997, Kim et al 2010b).

Image-space resolution recovery in LMOSEM

The LMOSEM reconstruction algorithm, which iterates the forward and backward projections in a given subset of measured listmode data, B_i , was adopted for the computational speed and incorporation of the elaborate system model (Parra and Barrett 1998, Wilderman et al



Figure 3. Thickened and tilted cones due to (A) angular and (B) positional measurement uncertainties in a Compton camera.

1998, Hudson and Larkin 1994, Kim *et al* 2010a). To compensate for the degradation in the spatial resolution from angular and positional uncertainties, we investigated for this paper a PSF model which is incorporable into statistical reconstruction as a part of the system matrix. The SV PSF (κ) applied in the computation of the forward and backward projections by image-space convolution operation (Reader *et al* 2003), is described by:

$$\boldsymbol{f}_{i}^{m,l+1} = \frac{\boldsymbol{f}_{i}^{m,l}}{\{\boldsymbol{S} \otimes \boldsymbol{\kappa}\}_{i}} \{\boldsymbol{c}^{m,l} \otimes \boldsymbol{\kappa}\}_{i}$$
(1)

where $f^{m,l}$ is the image to be reconstructed at the *m*th iteration from the *l*th subset (B_l) of the measured listmode Compton camera data. The sensitivity image, s_i is the probability that a photon emitted from the *i*th voxel is detectable by the pair of Compton detectors. For this study, we calculated the sensitivity image by surface integration over all measurable cones.



Figure 4. (A) Seven point sources located at different distances (3 to 9 cm spaced 1 cm apart) in front of a Compton camera, and (B) a graph of the measured spatial resolution of each point source against distance from the Compton camera.

 $c^{m,l}$ is the error image between the current estimated image and the real source distribution. It can be calculated by back-projecting the difference between the measured and estimated Compton data as follows;

$$\boldsymbol{c}_{i}^{m,l} = \sum_{j \in B_{l}} \frac{\boldsymbol{H}_{i,j}}{\sum_{p} \{\boldsymbol{f}^{m,l} \otimes \boldsymbol{\kappa}\}_{p} \boldsymbol{H}_{p,j}}.$$
(2)

The geometric system matrix (*H*) is expressible in terms of the intersecting lengths of the measured conical surfaces with voxels. *H* in equation (2) is implemented by the ray-tracing method (RTM) described in (Kim *et al* 2007). The measured listmode data were grouped into subsets ($\{B_l\}_{l=1,...,L}$) in chronological order. The symbol \otimes and $\{\}_i$ denote respectively the convolution in image space and the *i*th element of the vector which is obtained from the computation in the bracket.

Since the PSF modeling of positional (Δd) and angular (Δr) uncertainties in the Compton data space (projection space) has significant complexity and computational burden, our study considered using the image space convolution procedure with SV PSF over the image FOV. In figures 5(A) and (B) show the limited spatial resolution resulting from Δd and Δr , respectively. As the combined blurring effects from the tilted and thickened cones affect the reconstructed image (figure 5(C)), a PSF model in the image space can express all uncertainty components during Compton camera detection.



Figure 5. Reconstructed images and resolution profiles from Compton data showing effects from uncertainties due to (A) position, (B) angular orientation, and (C) both. (The Compton data is from a point source at a distance of 6 cm from the Compton camera in figure 4(A)).

Three different shift-variant PSFs

The kernel κ represents the PSF of the Compton camera and its variant between voxels. In equations (1) and (2), the convolution within the bracket with the SV kernel can be expressed as:

$$\{f \otimes \kappa\}_i = \sum_{n \in \{N_i\}} f_n \kappa_{i,}(x_n)$$
(3)

where N_i is the set of neighbors of voxel *i*.

We considered three different functions to model resolution degradation: Gaussian, Cauchy, and general parametric functions (equations (4)–(6)) for the SV PSF (κ) in LMOSEM (Ling et al 2008):

$$\kappa_i(\mathbf{x}) = A_i \cdot \exp\left[-\left(\frac{x - b_i}{c_i}\right)^2\right]$$
(4)

$$\boldsymbol{\kappa}_{i}(\boldsymbol{x}) = \boldsymbol{A}_{i} \cdot \left[\frac{c_{i}^{2}}{(x - b_{i})^{2} + c_{i}^{2}} \right]$$
(5)

$$\kappa_i(\mathbf{x}) = \mathbf{A}_i \cdot \left[\mathbf{1} + \frac{(x - b_i)^2}{\gamma_i c_i^2} \right]^{-\gamma_i}.$$
(6)

Each PSF at the *i*th voxel in the image space is characterized by either three or four characteristic parameters; A, b, c, and γ . A and b are the respective height and location of the peak for each function. The half-width at half-maximum (HWHM), c, controls the width of the function. The general parametric function in equation (6) has a general form between the Gaussian and Cauchy functions by the parameter of γ . If γ equals 1, the function reverts to a Cauchy function; if γ goes to infinity, it reaches a Gaussian function. In addition, the parameters, c and γ characterize the asymmetry of the PSF curve:

$$c_{i} = \begin{cases} c_{i;\text{left}}, & x < b_{i} \\ c_{i;\text{right}}, & x \ge b_{i} \end{cases}$$
(7)



Figure 6. Fitted Gaussian (red), Cauchy (blue), and general parametric (green line) curves to measurements (circle) from point sources at a distance of (A) 1 cm and (B) 11 cm from the Compton camera along the *x*-axis. Top: set-up of point source and Compton camera; middle: *x*-dependent of profiles of the three fitted curves; bottom: *z*-dependent of profiles of the three fitted curves.

$$\boldsymbol{\gamma}_{i} = \begin{cases} \boldsymbol{\gamma}_{i:\text{left}}, & \boldsymbol{x} < \boldsymbol{b}_{i} \\ \boldsymbol{\gamma}_{i:\text{right}}, & \boldsymbol{x} \ge \boldsymbol{b}_{i} \end{cases}$$
(8)

which is dependent on the distance from the Compton camera. Left and right parameterization can define the asymmetric shape of the PSF. The left indicates the part of the PSF from the peak (b) in the negative direction on the axis, whereas the right indicates the part of the PSF located in the positive direction.

Figure 6 shows the fitted curves of the three PSFs (red line: Gaussian, blue: Cauchy, green: general functions) for two point sources located at different distances. As the point source moves away the Compton camera, the spatial resolution becomes increasingly degraded and the PSF asymmetry is more prominent. The general function curve is almost similar to the

Gaussian curve when the point source is close to the Compton camera, but the curve changes to one intermediate between a Cauchy and a Gaussian functions as the point source moves away from the Compton camera.

PSF estimation using Monte Carlo simulation

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To obtain a SV-PSF over the FOV, Compton scattered data from distributed point sources were generated individually using the Geant4-based MC simulation software (Lee et al 2009). We generate Compton camera data for three pairs of scattering and absorption detectors placed on the x, y, and z axes, as in figure 7(A). Using this system configuration, we can guarantee data acquisition over a wide angular coverage. For each Compton camera, the scatterer and absorber detectors were simulated respectively by one double-sided silicon strip detector (DSSD) and 4 cadmium zinc telluride (CZT) detectors. A 5 cm spacing existed between the DSSD and CZT detectors. We simulated conditions found in realistic settings during measurements in which Compton data are affected by segmented position determination, limited energy resolutions, and Doppler broadening. In the simulation, the DSSD has 16×16 segments (pitch = 3.125 mm) over an area of 5 \times 5 cm², a thickness of 1.5 mm, and an energy resolution of 20 keV. Each CZT has 8 \times 8 segments (pitch = 3.125 mm) over a 2.5 \times 2.5 cm² area, a 5 mm thickness, and a 3% energy resolution. During simulation, a 10% energy photo-peak window was applied to the sum of the measured energies in the scatterer and absorber detectors. The Compton scattering angle was computed using incident photon and scattered electron energies. In the simulation, only events involving successive interactions in scatter and absorber detectors were accepted. If incident photons underwent multiple scattering interactions in detectors, events were discarded. Assuming the FOV is $10 \times 10 \times 10 \text{ cm}^3$ (the origin being located 6 cm from each Compton camera), we separately simulated processes from various point sources (with 10⁶ events per point, diameter of 1 mm, 140 keV). Sources were placed at different radii (r = 0 to 5 cm at steps of 1 cm) and different polar angles ($\varphi = 0$ to 360° with intervals of 22.5°) on the yz-plane, as depicted in figure 7(B). It was assumed that the PSFs corresponding to the open symbols could be obtained by reflecting those of solid symbols at the same radius in the yz-plane with respect to the diagonal line at 135° (or 315°) in figure 7(B). Thus we only performed a simulation sequence for 46 points indicated by solid symbols. The same simulation on the yz-plane, as shown in figure 7(B), was repeated at different distances (d = 1 to 11 cm at 1 cm intervals) from the surface of the DSSD on the x-axis (figure 7(A)).

To reconstruct the simulation data from each point source, we executed the LMOSEM algorithm (using the simple geometric system matrix) with 50 subsets and 3 iterations followed by Gaussian post-filtering (FWHM = 4.5 mm). The SV parameters (A: peak value, b: peak location, c: HWHM, and γ) of the three PSF curves as described in equations (4)–(6) were estimated separately from the x, y, and z-profiles of the reconstructed images using nonlinear least-squares. The control parameter, γ , of the general parametric function was limited within a range between 1 and 100. The PSF curves at every given voxel were determined from the parameters estimated from the neighboring point sources by piecewise linear interpolation. To compare the precision of the fit using the three different PSF curves, we calculated the sum of the squared 2-norm of the residual (SSR) and the Akaike information criterion (AIC), defined as follows:

$$SSR = \sum_{n=1}^{N} \left(\kappa_n^m - \kappa_n^e \right)^2 \tag{9}$$

$$AIC = Nln(SSR) + N_P \tag{10}$$



Figure 7. (A) A Compton camera system consisting of three scattering and absorbing detector pairings placed in a mutually orthogonal configuration. (B) Set-up illustrating the sequence of simulations of point sources located at different radii and polar angles in the yz plane and the different positions along the *x*-axis.

where κ_n^m and κ_n^e are the respective measured and estimated *n*th points on the PSF curve. The AIC is determined from both the fitness accuracy (SSR) and the model complexity (parameter number, N_P).





Figure 8. (A) SSR and (B) AIC comparisons of the three PSF functions: Gaussian, Cauchy, and general parametric curves.

Results

Shift-variant PSF estimation

In this paper, we proposed three different functions-Gaussian, Cauchy, and general parametric-to develop expressions for the SV-PSF of the Compton camera. The parameters for each model were determined by simulations according to the point source placement strategy, as indicated in figure 7. In figure 8, we compared the PSFs of the three models in terms of SSR and AIC; the SSR represents the error between the measured and the fitted curves whereas the AIC is a measure of the relative suitability of the fit with a penalty for the parameter number to be estimated. The general parametric model, having an additional parameter, resulted in the lowest SSR and AIC although the two parameter values of this model were not much different from those of the Gaussian model. The Cauchy model performed poorly in terms of fitness accuracy.

Figure 9 shows representative samplings of the estimated HWHMs of the x-profile for the Gaussian (row (A)) and the general parametric (row (B)) PSFs. For each row, the blue and red plots are the respective left and right curves of the corresponding PSFs. The HWHMs (c_{left} and c_{right}), which describe the asymmetric width of the PSF, varied over different radii and polar angles in the yz-plane and distances from the Compton camera along the x-axis. Figure 9(C)shows the asymmetries ($c_{left}-c_{right}$) of these estimated HWHMs over the FOV.

Figures 10 and 11 show the distance dependence of the scatter plots of the peak location (b, relative to each point source position) and the asymmetric γ parameters (γ_{left} and γ_{right}). In each plot, the triangles for a given distance are the PSF parameter values estimated from point sources at different radii and polar angles presented in figure 7(B). Figure 10 presents the values of the SV and non-zero peak location parameters for the Gaussian and general parametric PSF models (rows (A) and (B), resp.). A non-zero peak location indicates the skewness of the corresponding PSF for given a point source. The γ parameter in the general parametric model provides flexibility in the shape of the curve between that for a Gaussian and Cauchy functions and a measure of the asymmetry ($\gamma_{\text{left}} - \gamma_{\text{right}}$). The high γ parameter values in figure 11 generate PSFs with long tail-like Cauchy functions.

Resolution recovery reconstruction with three SV-PSFs

The estimated asymmetric Gaussian and general parametric functions were incorporated into LMOSEM of equation (2) as a SV- PSF (κ); these are denoted here by SV-Asym-Gauss



Figure 9. Plots of the estimated HWHM (blue: c_{left} , red: c_{right}), which controls the curve width of (A) the Gaussian and (B) general parametric PSFs and is SV over the FOV, from point source measurements at different distances, polar angles, and radii; (C) asymmetry (c_{left} – c_{right}) plots of the left and right HWHMs for Gaussian (left) and general parametric (right) PSFs.

and SV-Asym-General, respectively. Each LMOSEM with SV-PSF was compared with the shift-invariant and symmetric Gaussian (SIV-Sym-Gauss) function as well as the standard LMOSEM followed by post smoothing (LMOSEM-Post-Gauss). The SIV-Sym-Gauss was developed using the measured FWHMs at the origin in the FOV. The MC simulation was performed with a single point source that was positioned at 3, 6, and 9 cm from the Compton camera on the *x*-axis, as sketched in figure 12(A). All LMOSEMs were performed with 30 subsets and 10 iterations for 10^6 MC events. In figures 12(B) and (C) show for successive iterations the resolution recovery rate on the *x*-axis and *yz*-plane, respectively. All resolution relative to the standard LMOSEM. Comparing with SIV-Sym-Gauss, the SV-Asym-PSFs yielded the more uniform resolution recovery rate over the different positions in the FOV.



Figure 10. Scatter plots of the peak location (*b*) parameters for (A) the Gaussian and (B) general parametric PSFs at different distances from the Compton camera situated on *x*-axis (left: *x*-profile, middle: *y*-profile, right: *z*-profile).



Figure 11. Scatter plots relating to the γ parameter, which determines the shape of the curve for the general parametric PSF: (A) left and (B) right curves, and (C) the asymmetry ($\gamma_{\text{left}} - \gamma_{\text{right}}$) over different distances (left: *x*-profile, middle: *y*-profile, right: *z*-profile).

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Figure 12. (A) Set-up of point source for simulations for three different positions: (x = 3, y = 0, z = 0), origin (0, 0, 0), and (-3, 0, 0); (B) FWHM plots along the *x*-axis; (C) mean FWHM plot along the *y*- and *z*-axes; LMOSEM followed by a Gaussian smoothing filter (Post-Gauss), LMOSEM with shift-invariant and symmetric Gaussian PSF (SIV-Sym-Gauss), LMOSEM with SV and asymmetric Gaussian and general PSF (SV-Asym-Gauss and SV-Asym-General).

Although SIV-Symm-Gauss provided the best resolution recovery rate at the closest distance (d = 3 cm) from the Compton camera, its recovery rate worsened more than that of SV-Asym-PSFs with distance (d = 6 and 9 cm). Among the SV-Asym-PSFs, SV-Asym-Gauss showed the better resolution recovery rate than SV-Asym-General.

Using the MC data with a point source placed at the origin in the FOV, we analyzed the influences on the resolution recovery rate for different sampling intervals in the point-source placement strategy and the different ways to express each PSF (figure 13). We studied the point-source placement of figure 7(B) with different angular intervals of 45° and 22.5° (square versus circle symbols in figure 13) on the *yz*-plane, as well as symmetric versus asymmetric (triangular versus circle symbols in figure 13) SV-PSFs. In figure 13, solid and open symbols distinguish the PSFs expressed respectively by only the HWHM parameter (*c*) and by both the non-zero peak location (*b*) and HWHM parameters. There were slight differences with different sampling intervals and PSF expressions, but the combination of asymmetric PSF and the PSF expression of only the HWHM parameter (*c*) resulted in the best performance.

The combined MC data of two point sources were generated by simulating 300×10^6 photons from each point source. Two point sources, separated 4 cm apart, were located at different distances of 4 and 8 cm on the *x*-axis, as indicated in figure 14(A). In figures 14(B)–(E) show planes containing each point source of the reconstructed images by standard and resolution recovery LMOSEMs. In figures 15(A) and (B) are the respective



Figure 13. FWHM plots of resolution recovery reconstruction according to different PSF expressions and point source placement: symmetric versus asymmetric PSF and 45° versus 22.5° sampling intervals of polar angles in *yz*-plane. (Solid and open symbols indicate PSFs expressed by only HWHM and by both peak location and HWHM parameters, respectively.) (A) Gaussian and (B) general PSF functions. (Note that solid symbols overlapped with open ones in the Gaussian plot.)



Figure 14. Reconstructed images from simulation data of two point sources at (x = 2, y = 0, z = 0) and (-2, 0, 0) under 900 $\times 10^6$ emissions per point source; (A) simulation schematics, LMOSEM (B) without PSF model, (C) with SIV-Sym-Gauss, (D) SV-Asym-Gauss, and (E) SV-Asym-General PSFs.

measured spatial resolutions and normalized activity ratio of a pair of point sources. The standard LMOSEM in figure 14(B) gave insufficient spatial resolution. Figures 14 and 15(A) show that both the SIV-Sym- (figure 14(C)) and SV-Asym-PSF (figures 14(D) and (E)) reconstructions provided more improved spatial resolutions qualitatively and quantitatively



Figure 15. Plots of (A) measured resolutions on x-axis and yz plane (the numberings 1 and 2 correspond to point sources at distances of 4 and 8 cm from the Compton camera, respectively.), and (B) normalized activity ratio between two point sources.

than the standard LMOSEM (figure 14(B)). Resolution recovery reconstructions improved the spatial resolution by about a factor of 2 in terms of FWHM. Compared with SIV-Sym-Gauss, SV-Asym-PSFs resulted in greater recovered activity ratios between two point sources and uniform resolution recovery rates over different positions in the FOV.

The MC data $(1.08 \times 10^6 \text{ events})$ of six point sources, located as shown in the depictions on the far left column (A) of figure 16, were generated. The distance between two neighboring point sources is 2 cm. The *yz*- (first to third rows) and *xz*-planes (fourth to fifth rows) of the reconstructed images were displayed to include two or three point sources in the same plane. The third row corresponds to a more distant plane from the Compton camera than the first. Figures 16(B) and (C) are respectively LMOSEMs without and with SIV-Sym-Gauss. Figures 16(D) and (E) are LMOSEMs adopting SV-Asym-Gauss and SV-Asym-General PSFs, respectively. Comparing with the standard LMOSEM, all LMOSEMs with SIV-Sym- and SV-Asym-PSF models differentiated the distinct point sources better in all diagonal directions in the *yz*-plane. However, the recovery rate was still dependent on the distance from the Compton camera.

Discussion

In this study, we investigated LMOSEM with an image-space resolution recovery technique for the Compton camera. We proposed a placement strategy of point sources (figure 7) to measure PSF and considered three different curve fitting models-Gaussian, Cauchy, and general parametric functions-to estimate the SV PSF. From MC simulations of a single point source, the shift-variance and asymmetry of the three or four parameters of each PSF model were shown to be dependent on the different radii and polar angles on the yz-plane and distances along the x-axis (figure 9-11). In figures 9(C) and 11(C), the scattering degree of triangle symbols ($c_{\text{left}}-c_{\text{right}}$ and $\gamma_{\text{left}}-\gamma_{\text{right}}$) from the zero line (red dotted line) indicates the asymmetry variation of HWHM and γ parameters at different radii and polar angles for a given distance. In particular, the asymmetry variation of HWHMs showed a similar pattern over distance from the Compton camera for both Gaussian and general parametric functions. There was little difference between the fitness accuracy in terms of SSR and AIC for the Gaussian and general parametric models, despite the extra γ parameter available to the general parametric model. However, this parameter adds flexibility in shaping the curve between those for Gaussian and Cauchy functions and in representing the orientation and position dependent variation in the PSF.



Figure 16. Reconstructed images from simulation data of 2×3 point source array spaced 2 cm apart at different diagonal directions and distances (4, 6, and 8 cm) from the Compton camera situated on the *x*-axis; (A) simulation schematics, LMOSEM (B) without PSF model and with (C) SIV-Sym-Gauss, (D) SVAsym-Gauss, and (E) SV-Asym-General PSFs.

LMOSEMs adopting SV-Asym-PSF models yielded better recovered and uniform spatial resolution over the FOV than the standard LMOSEM and LMOSEM-SIV-Sym-Gauss (figure 12). For SV-Asym-PSFs, the Cauchy function yielded the worst fitness accuracy. There was little difference between Gaussian and general parametric functions in terms of fitness accuracy and resolution recovery rate; differences tended to depend on positional variations in the FOV.

For all PSF models, the kernel expression with only asymmetric HWHMs provided better resolution recovery rates. The angular sampling of 22.5° in orientation provided a more accurate SV-Asym-PSF expression for the Compton camera. The parameters and shapes of the PSF should be sensitive to different detector geometries, source energies, computation methods used to determine Compton scattering angle, and the possibility of reverse interaction ordering (that is, the first interaction in the absorber and then the second interaction in the scatterer). Changes in the detection process require a recalculation of the PSF (simulation and reconstruction time for obtaining all PSFs used in this study was ~ 8 months in total). However, the reconstruction time is unaffected by the recalculation since the resolution recovery adopts the pre-calculated PSFs. In future works, we can improve the computational efficiency by using parallel computing of multi-CPUs and GPU.

Conclusion

MC simulations were performed iteratively by moving a point source over various locations in the FOV of a Compton camera. Parameters of the SV-Asym-PSF were estimated from the simulated MC data of the point source. Three different curve-fitting functions—Gaussian, Cauchy, and general parametric models—were considered. In addition, the SV-Asym-PSF was incorporated into the accelerated resolution recovery reconstruction, LMOSEM. Although there was little difference in the fitness accuracy between Gaussian and general parametric models, the γ parameter of the latter provided flexibility in shaping the curve between those of the Gaussian and Cauchy functions in the FOV. All LMOSEM-SV-Asym-PSFs improved the spatial resolution by 2.6 times than the standard LMOSEM for MC data of one point source at origin (figure 12). For the simulation of two point sources, the resolution recovery reconstructions recovered more the spatial resolution by about a factor of 2 in terms of FWHM. Compared with LMOSEM-SIV-Sym-Gauss, LMOSEM-SV-Asym-PSFs resulted in higher recovered activity ratios for two point-source pairs at different positions in the FOV.

Acknowledgments

This work was supported by the Nuclear Research & Development Program of the National Research Foundation of Korea (NRF) (grant code: 2005-2004723, 2008-2003852, 2010-0026012) and NRF Korea grant (NRF-2010-355-E00045) funded by the Ministry of Education, Science & Technology (MEST) of Korea.

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